Statistical tools for analysing uncertainty in computer model predictions

Jeremy Oakley

School of Mathematics and Statistics

The University Of Sheffield.

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- Computer experiment: evaluating f at difference choices of x
  - A 'model run': evaluating f at a single choice of x.

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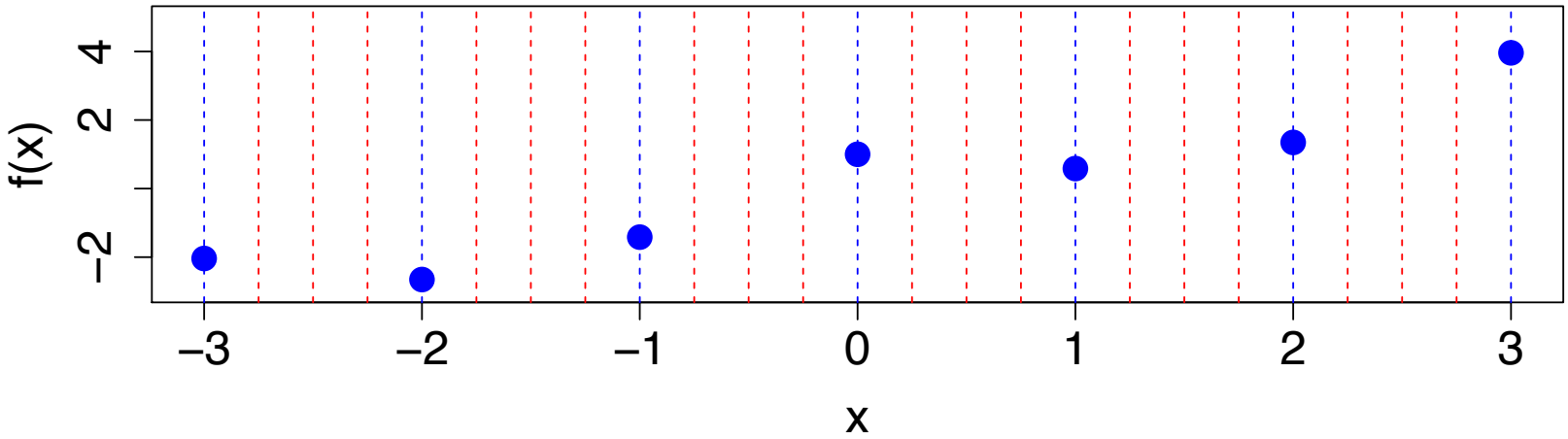
- What do we do if f is computationally expensive?

Computationally expensive models



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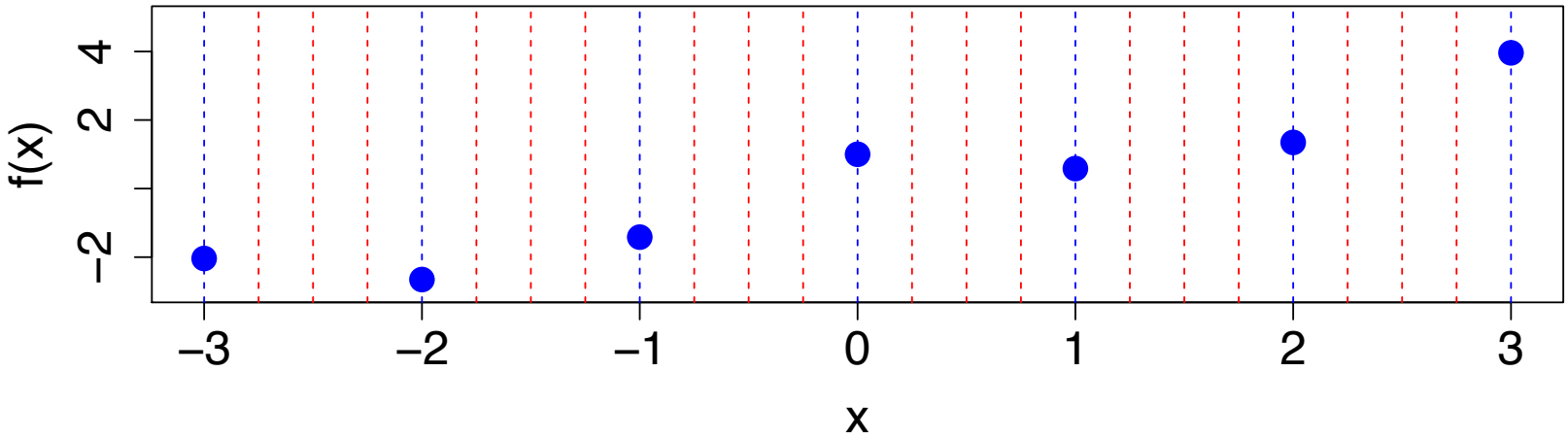

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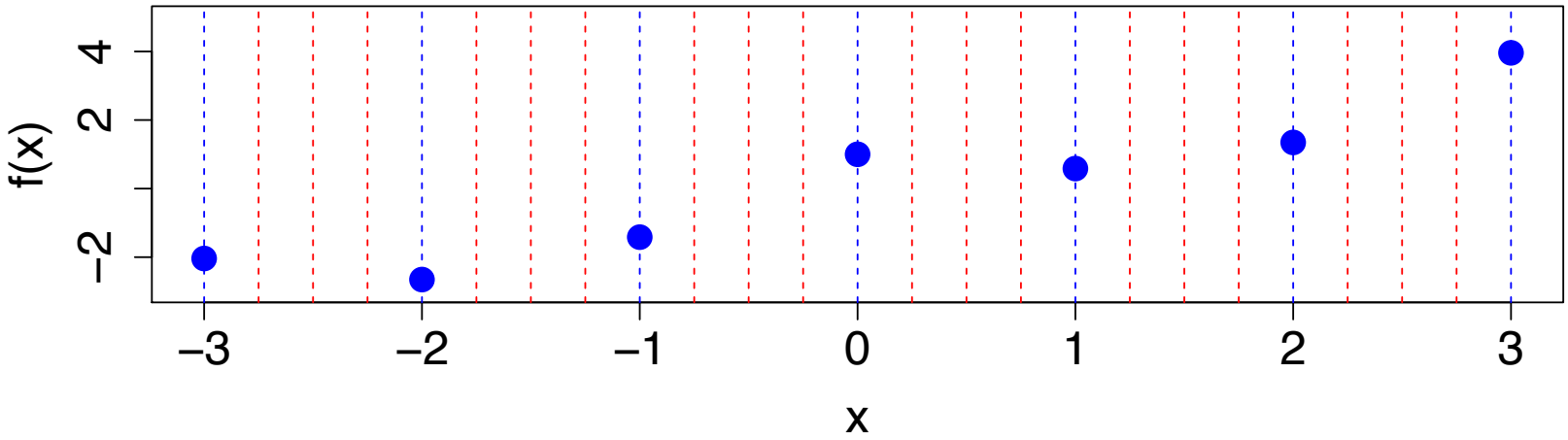

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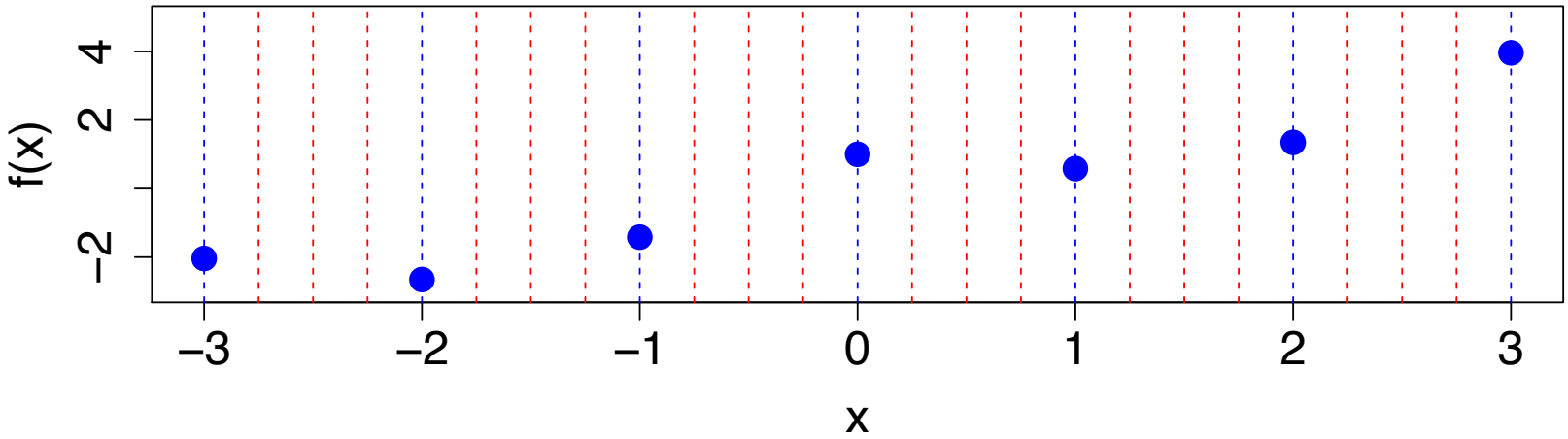

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- Popular technique: Gaussian process emulation (Sacks et al 1989)

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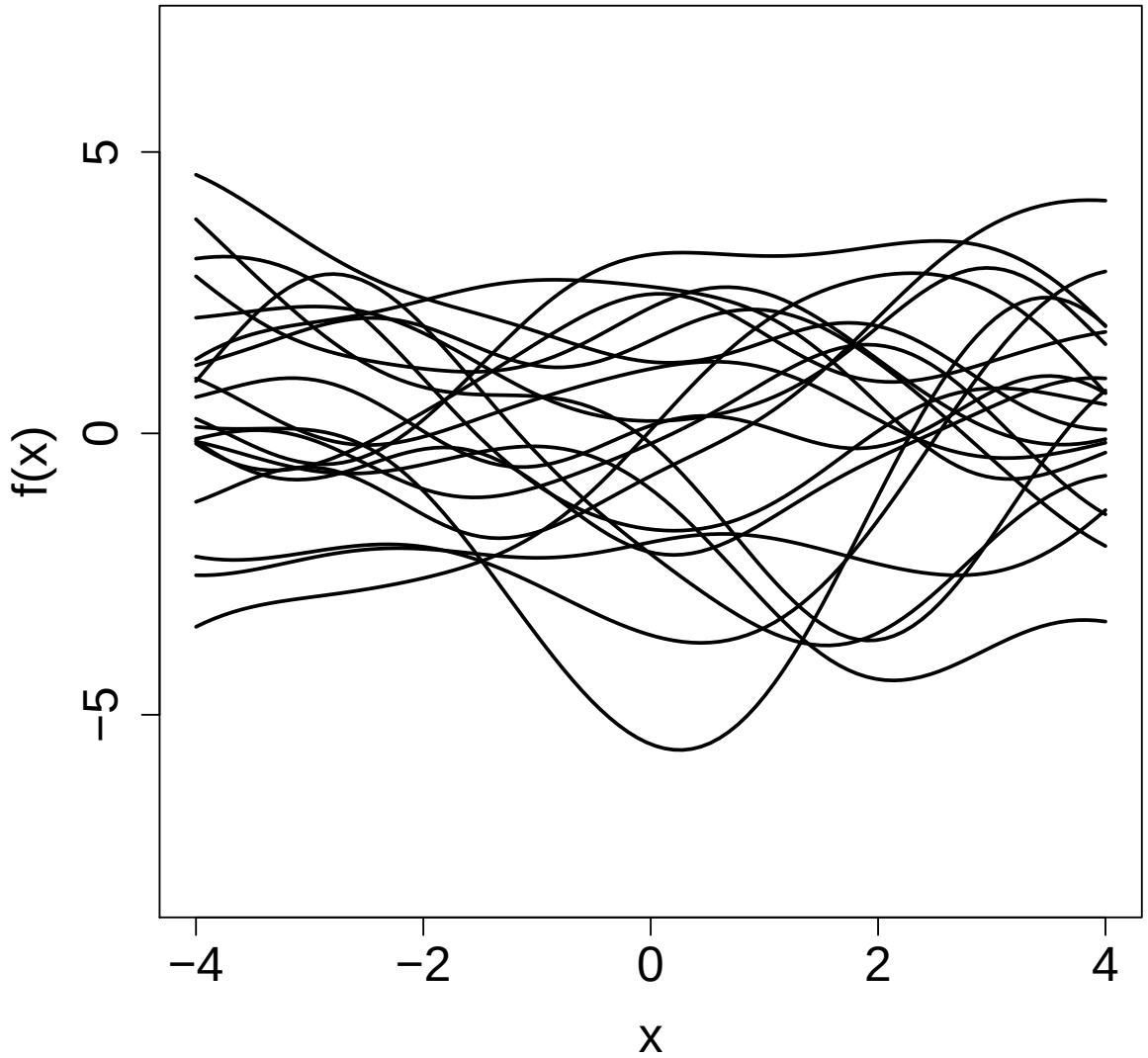

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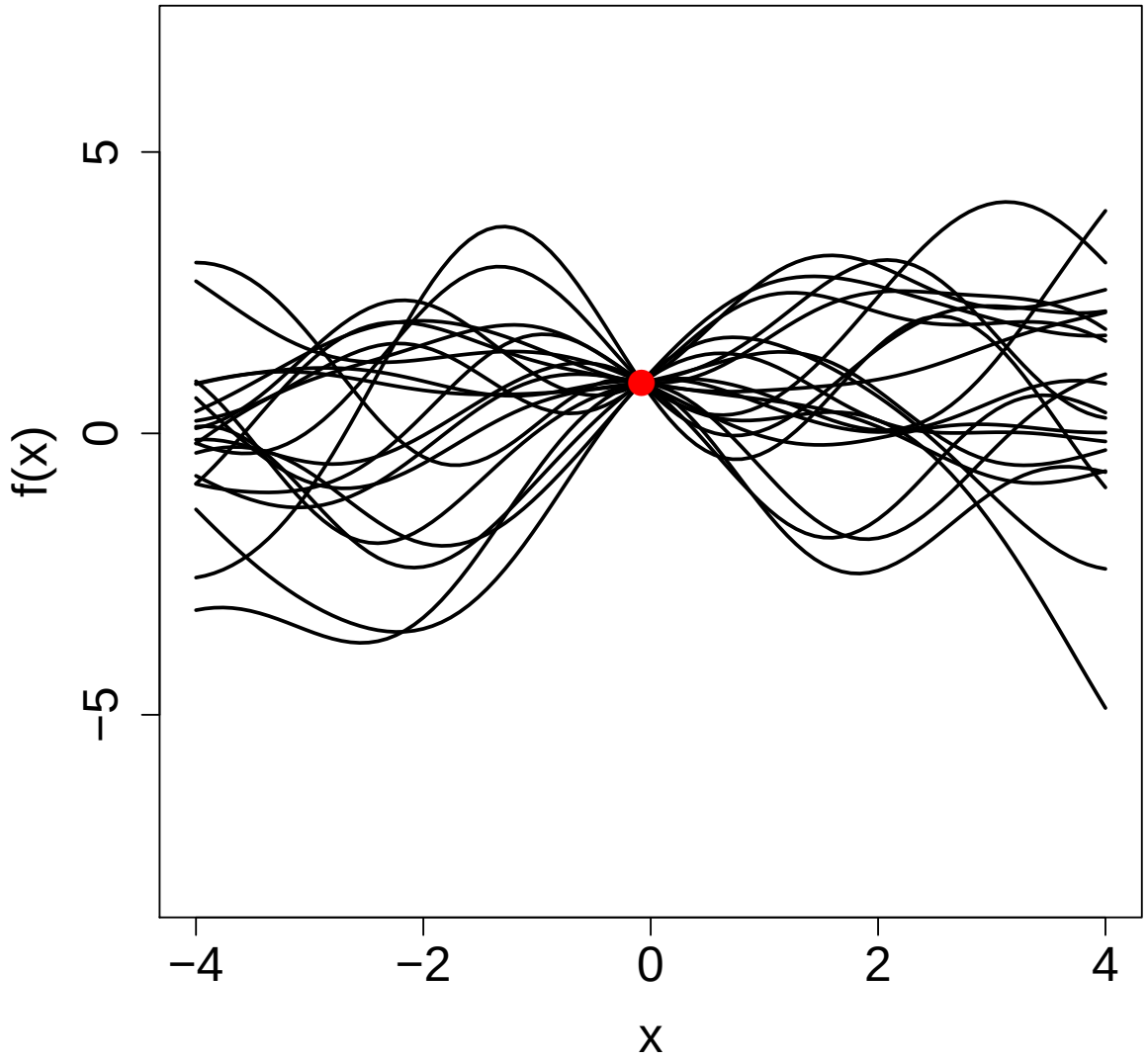

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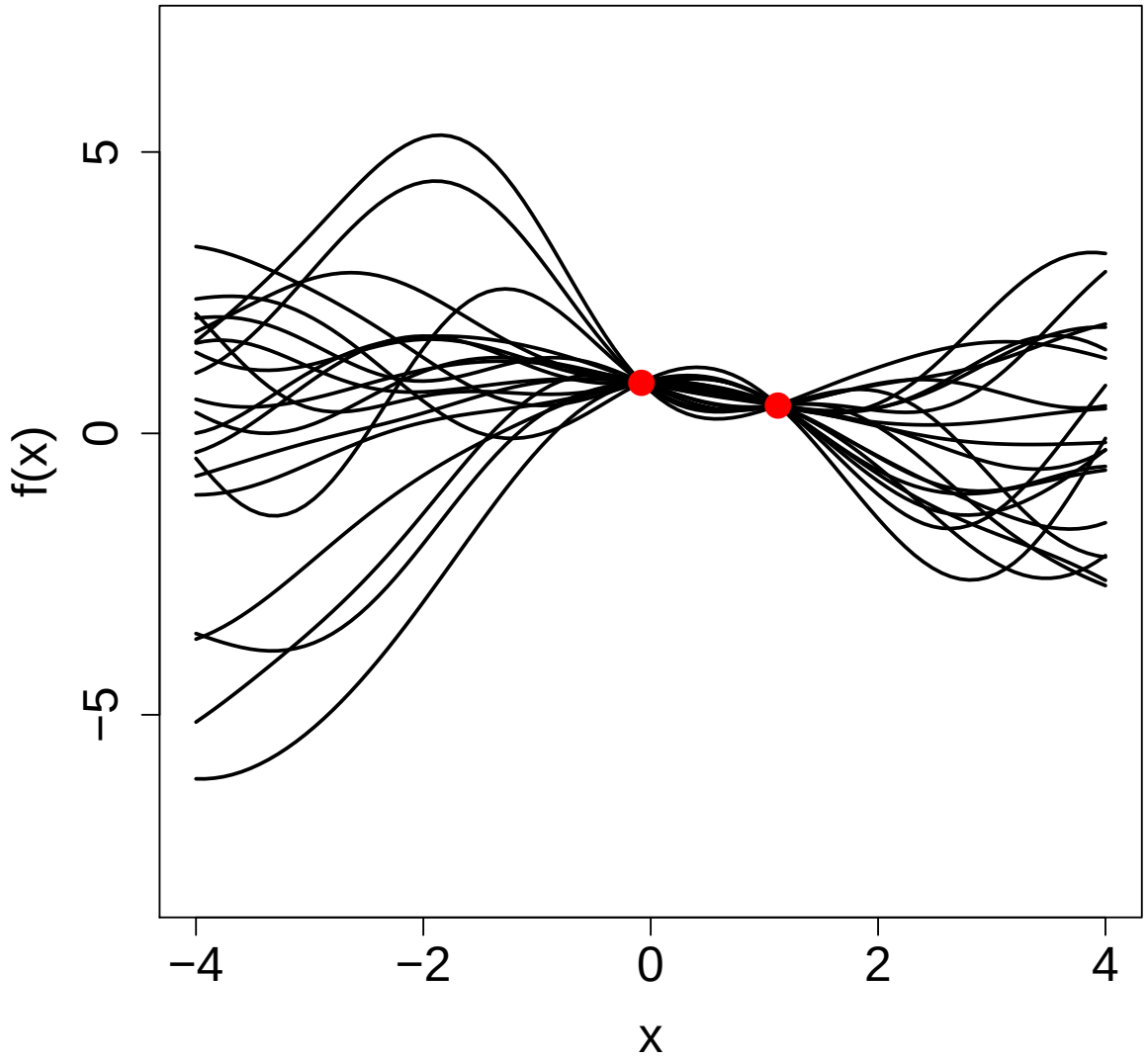

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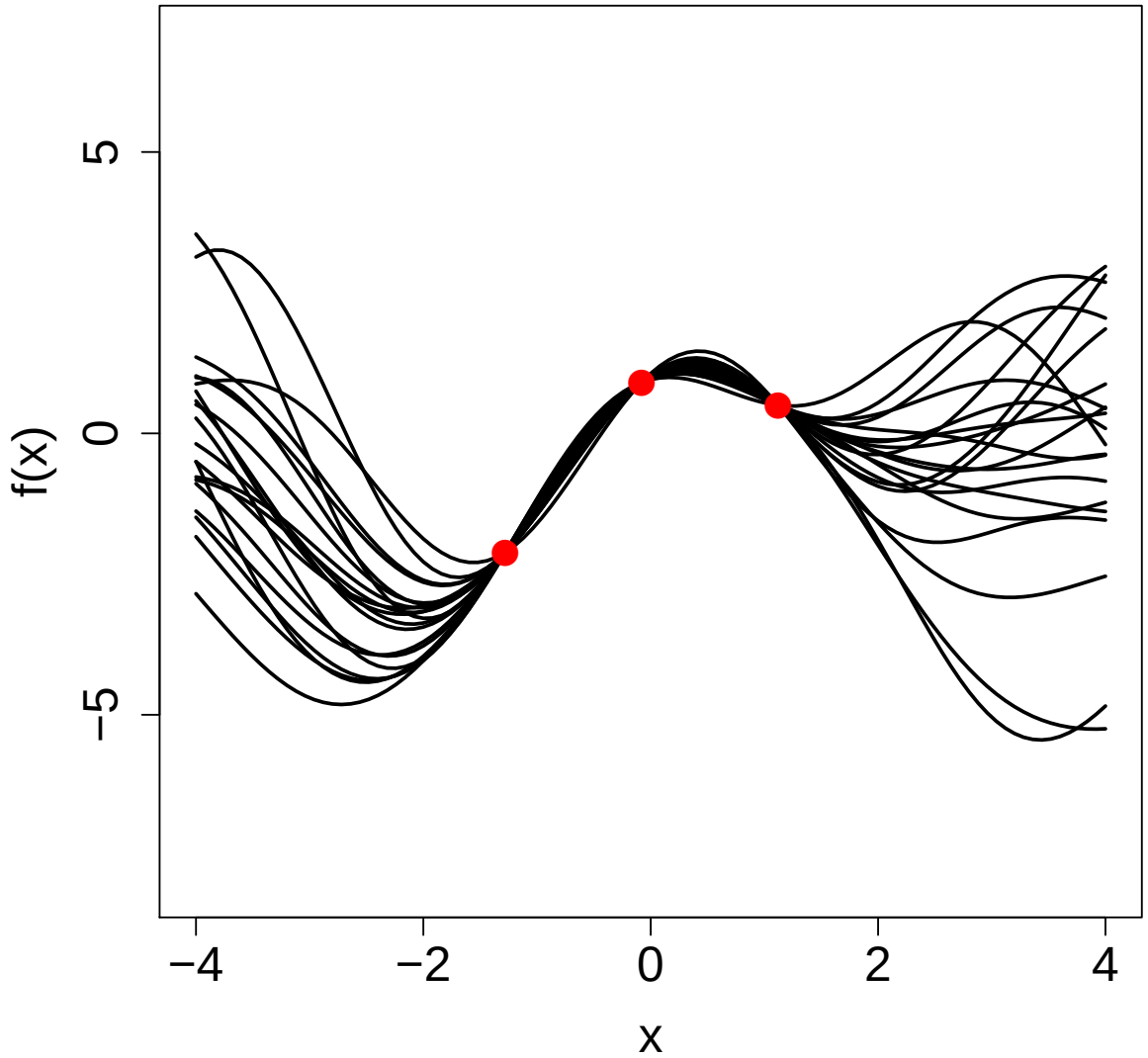

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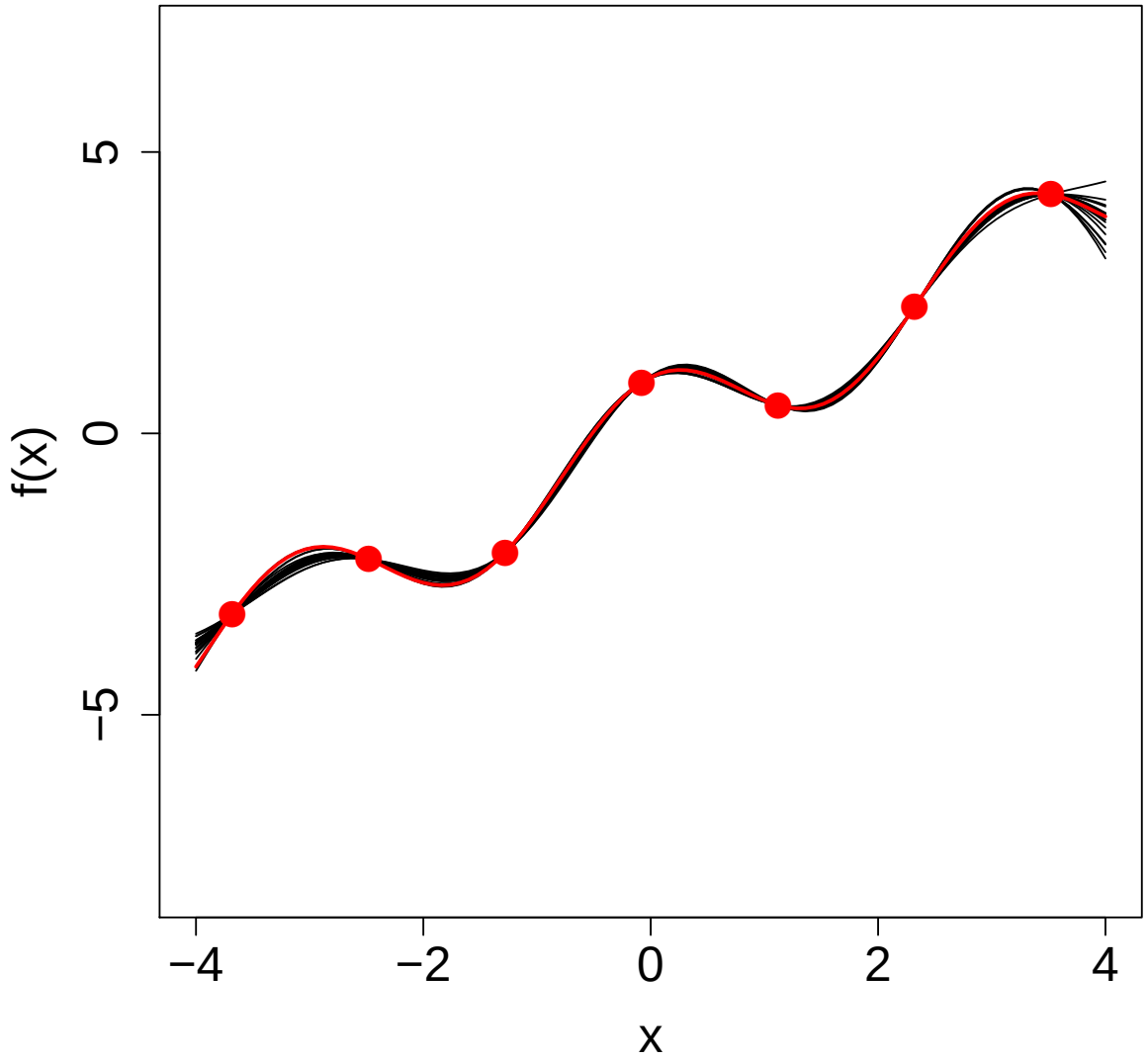

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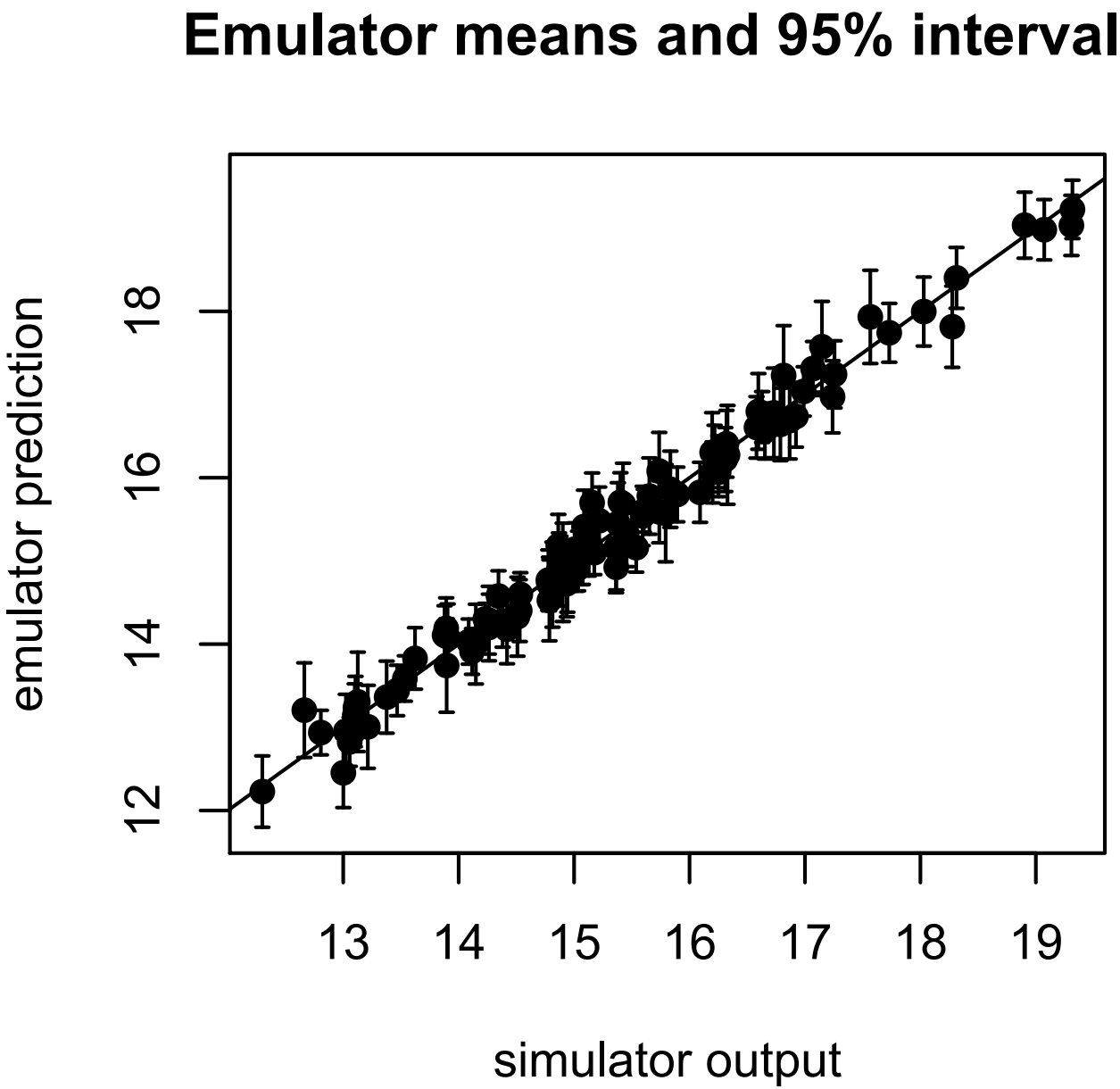

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- The expected reduction in variance if value of $\mathbf{X}_u$ is learnt, because

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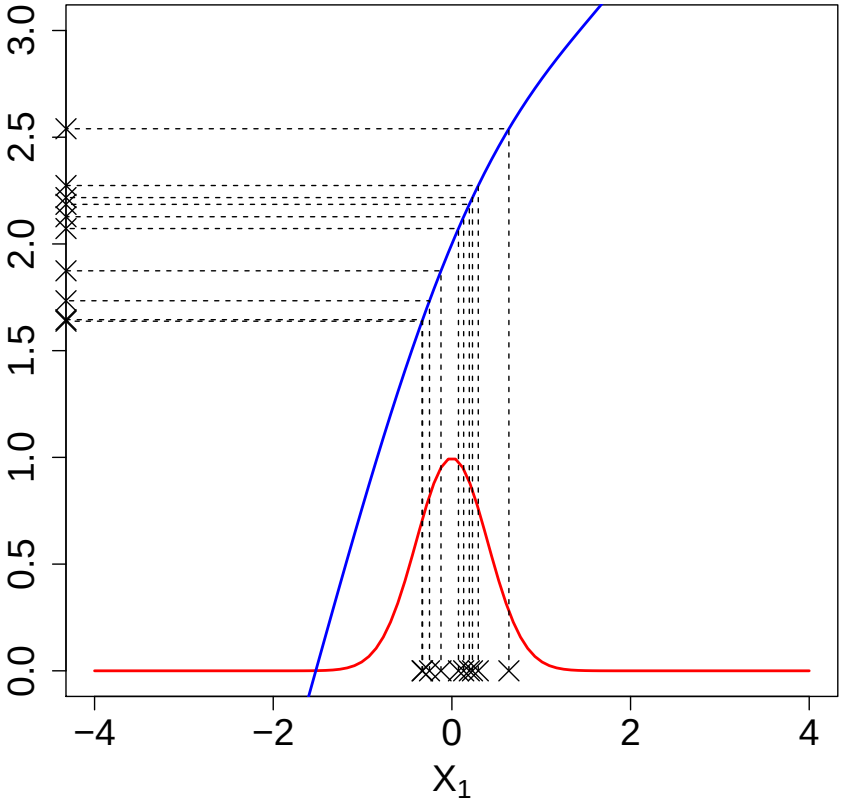
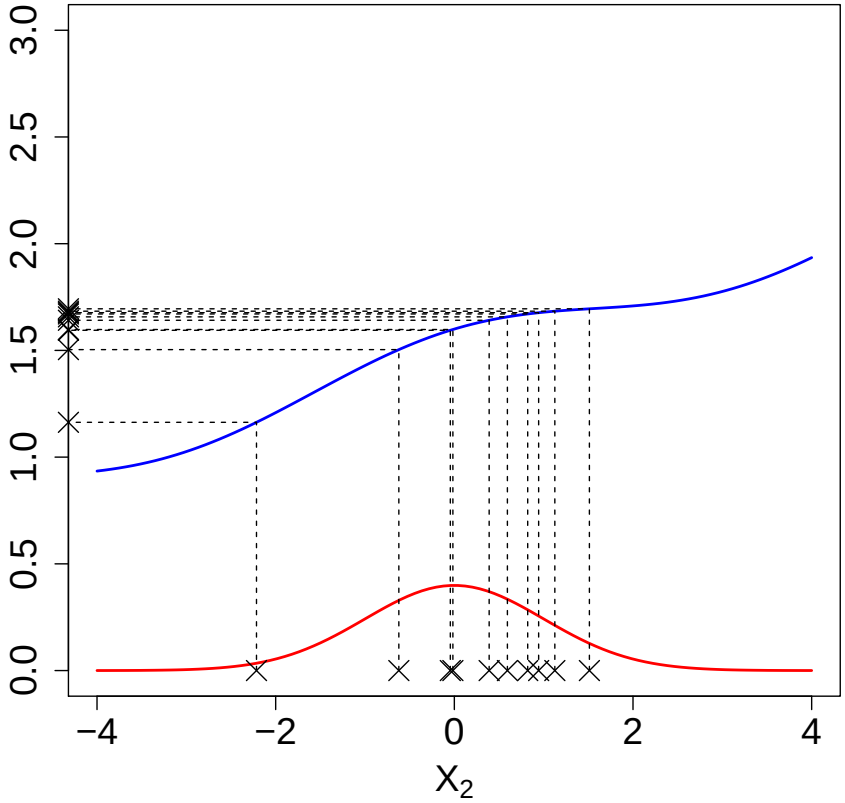
- Can speed up computation with emulator (Oakley & O'Hagan, 2004)

Variance-based sensitivity analysis





— $p_{X_1}(x_1)$

— $E(Y|X_1 = x_1)$

— $p_{X_2}(x_2)$

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[1] MUCM case study: analysis by John Paul Gosling, Hugo Maruri-Aguilar, Alexis Boukouvalas

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- GSK analysis investigated sensitivity of output to 9 inputs, using 8200 model runs
- We consider sensitivity of output to 20 inputs, using 340 model runs

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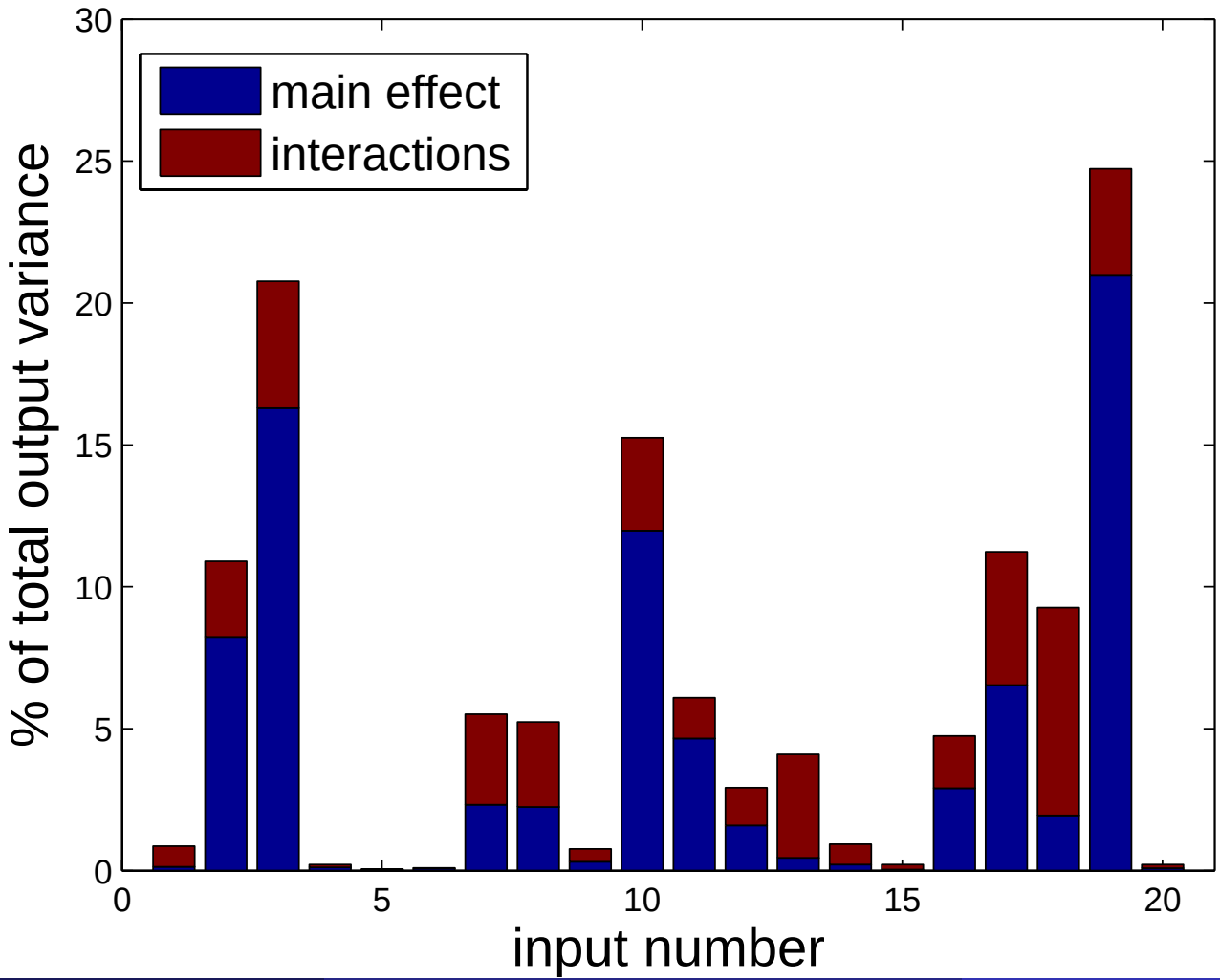
Variance based sensitivity analysis

Analysis for an individual output: no. of infections in 2-3 age group after 2 years



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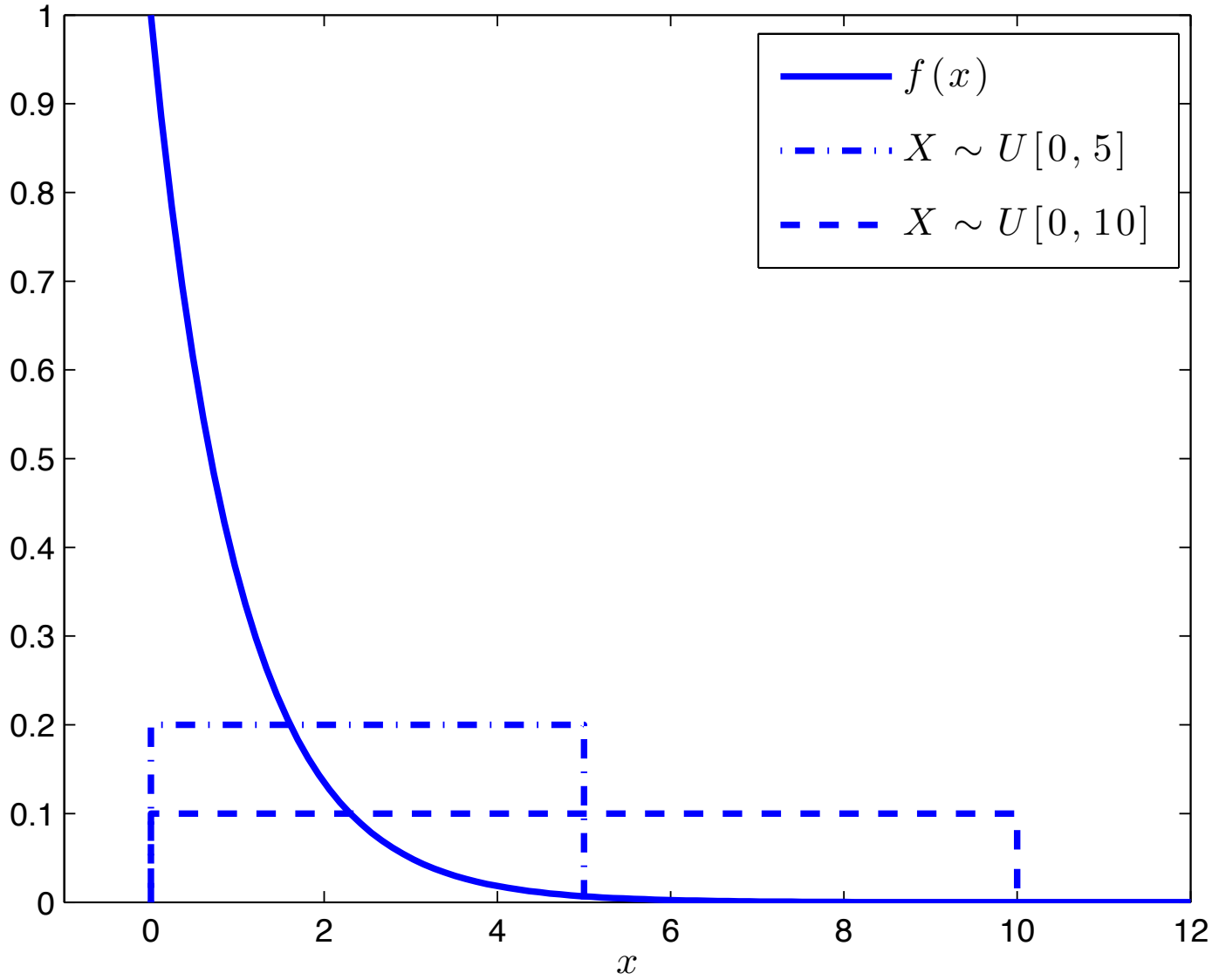
Increasing b increases the variance of X but *decreases* the variance of Y


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
x
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$X \sim U[0, 5]$
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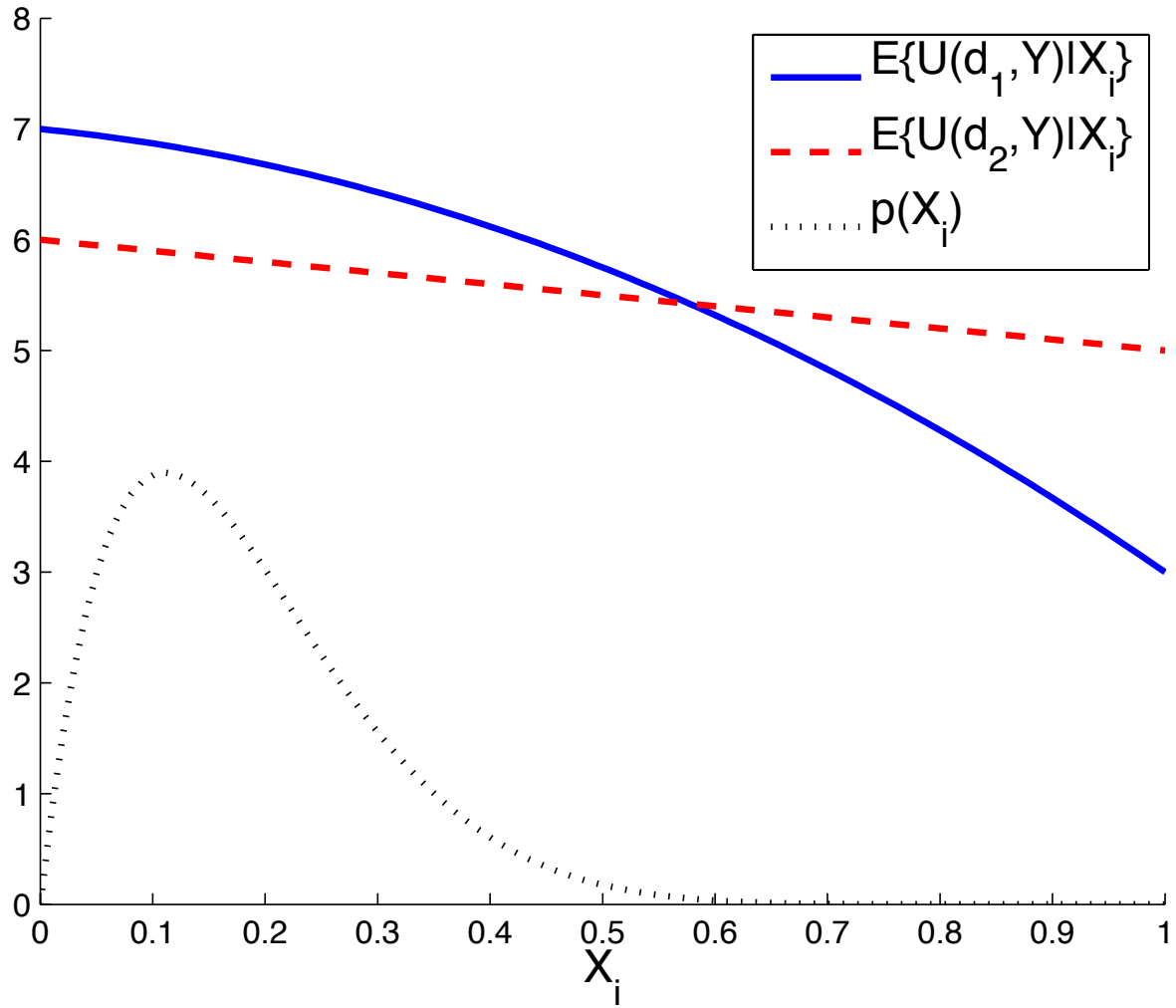
Sensitivity analysis for decision making



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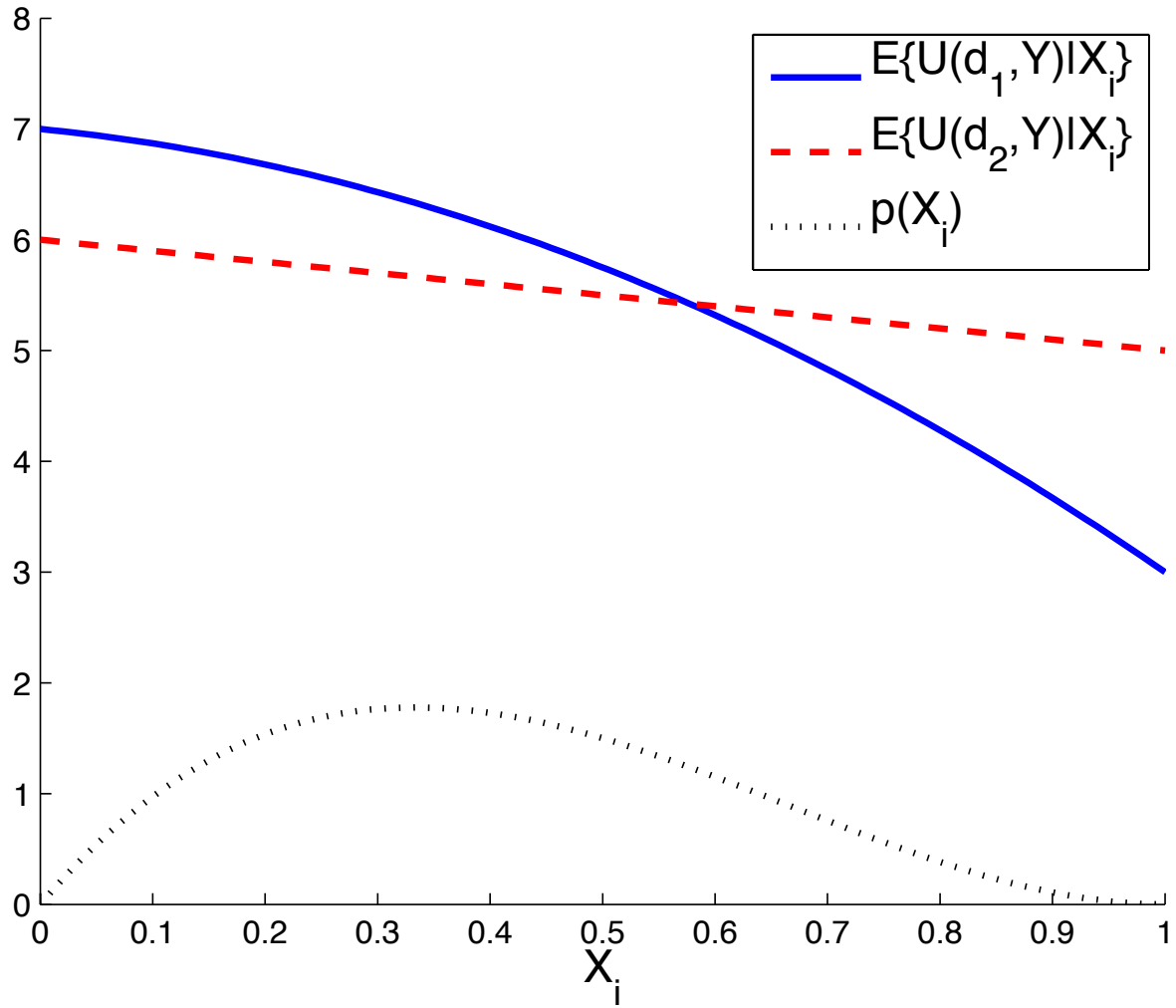

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- And what happens if the model is wrong?

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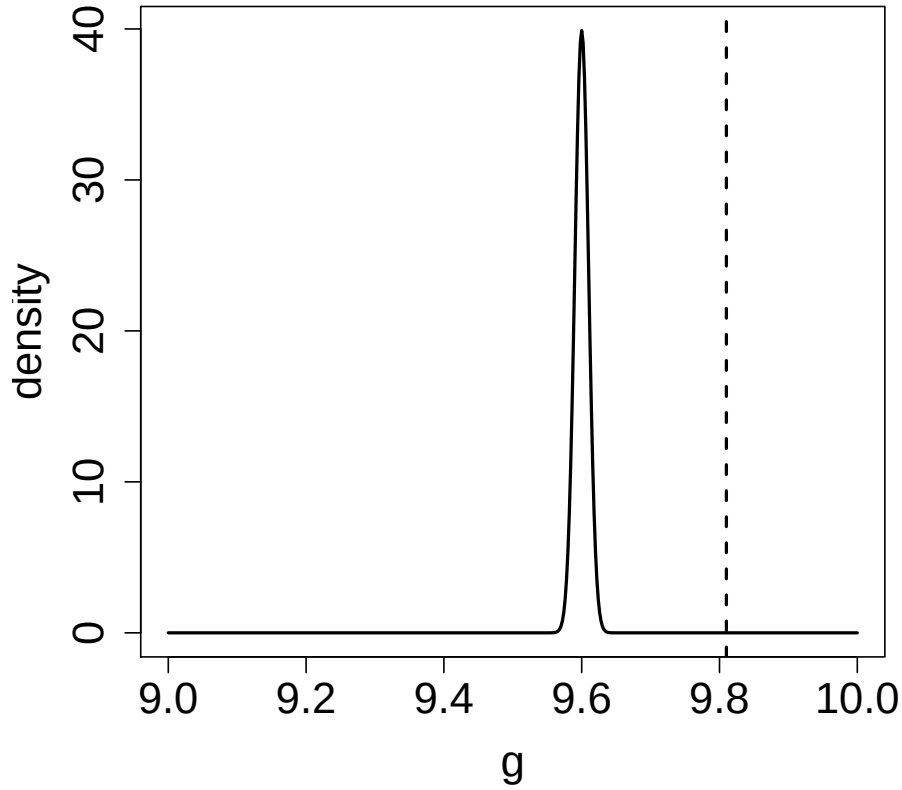

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- In some settings, can infer $\delta()$ from observations, but in others, (prior) judgements important

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$$I(x) = \frac{|z - E\{f(x)\}]|}{[Var\{f(x)\} + Var(\varepsilon) + Var(\delta)]^{1/2}}.$$

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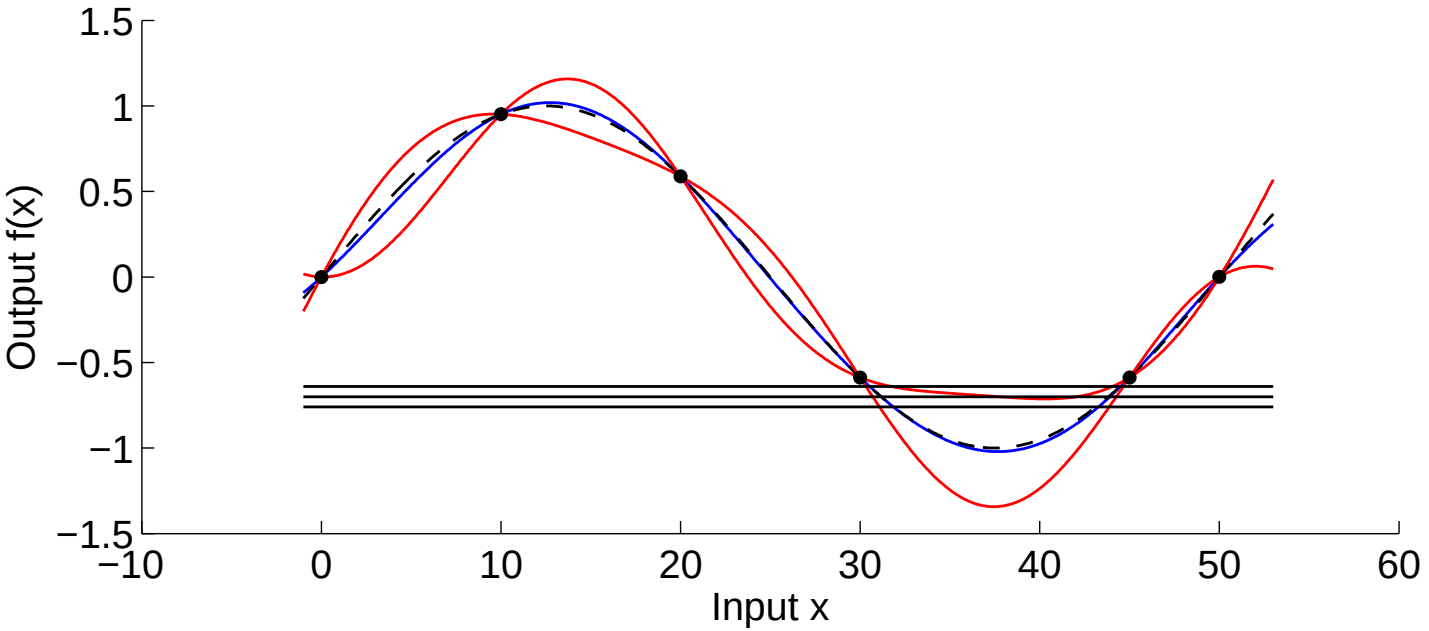
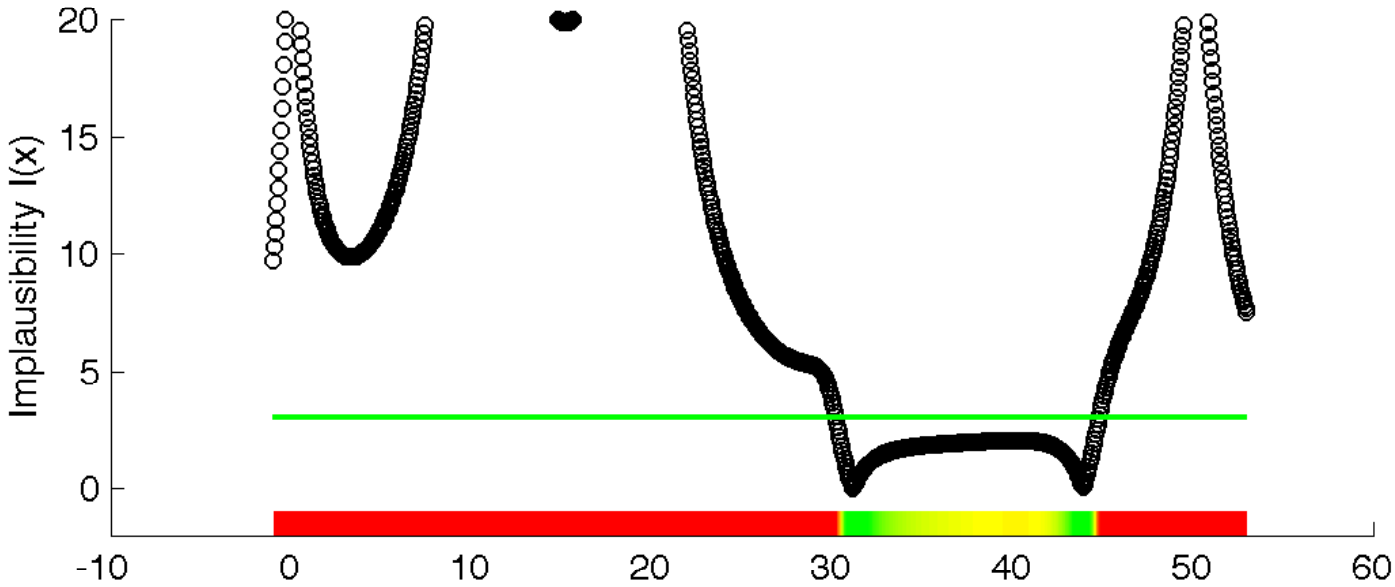
$$I(x) = \frac{|z - E\{f(x)\}]|}{[Var\{f(x)\} + Var(\varepsilon) + Var(\delta)]^{1/2}}.$$

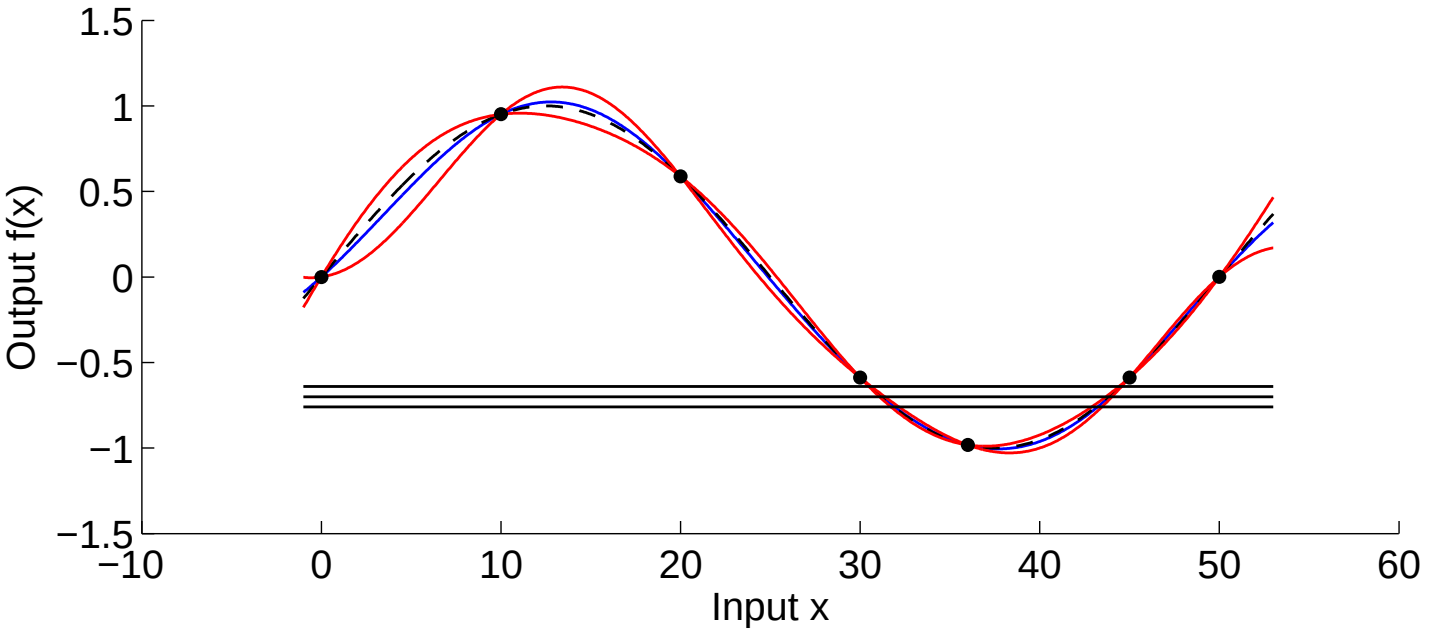
- Do further runs in the non-implausible region and rebuild emulator

Toy example


Output f(x)
Input x
1.5
1
0.5
0
−0.5
−1
−1.5
−10
0
10
20
30
40
50
60


Implausibility I(x)
20
15
10
5
0
-10
0
10
20
30
40
50
60

Toy example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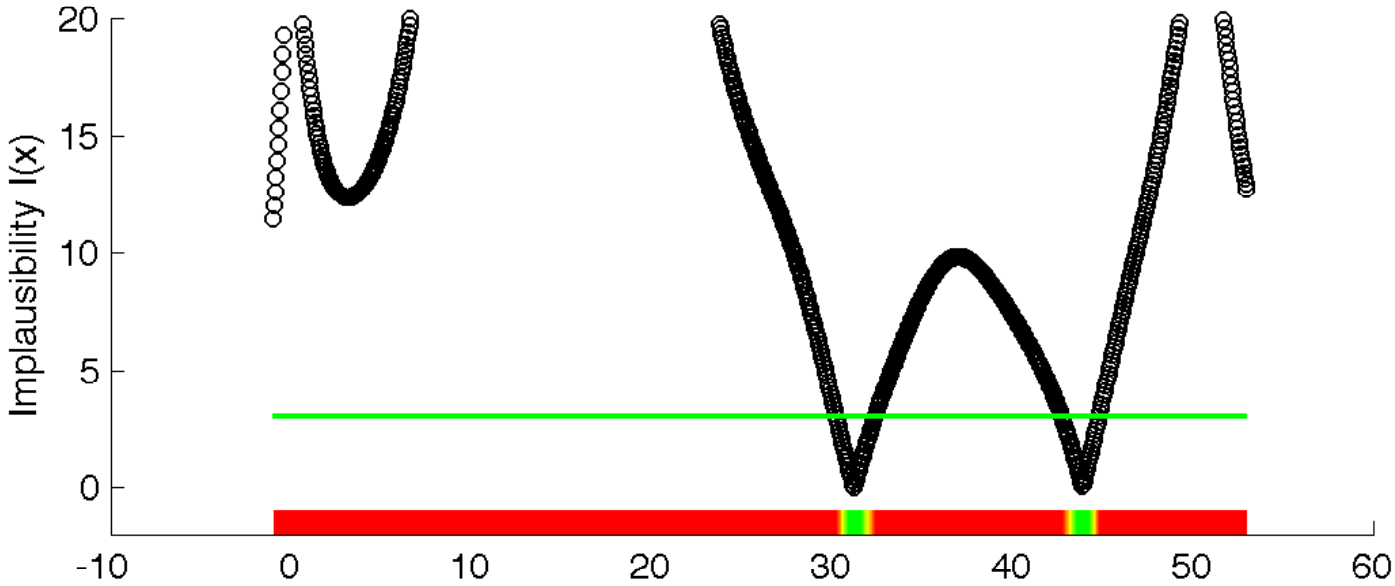
Implausibility I(x)
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10
5
0
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0
10
20
30
40
50
60

Case study[2]: HIV in Uganda

- "Mukwano": a dynamic, stochastic, individual based model that simulates sexual partnerships and HIV transmission
- Births, deaths, partnership formation and dissolution and HIV transmission were modelled using time-dependent rates

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- 22 inputs, e.g. proportions of men and women in "high sexual activity" groups, transition probability of HIV per sex act during primary stage of infection

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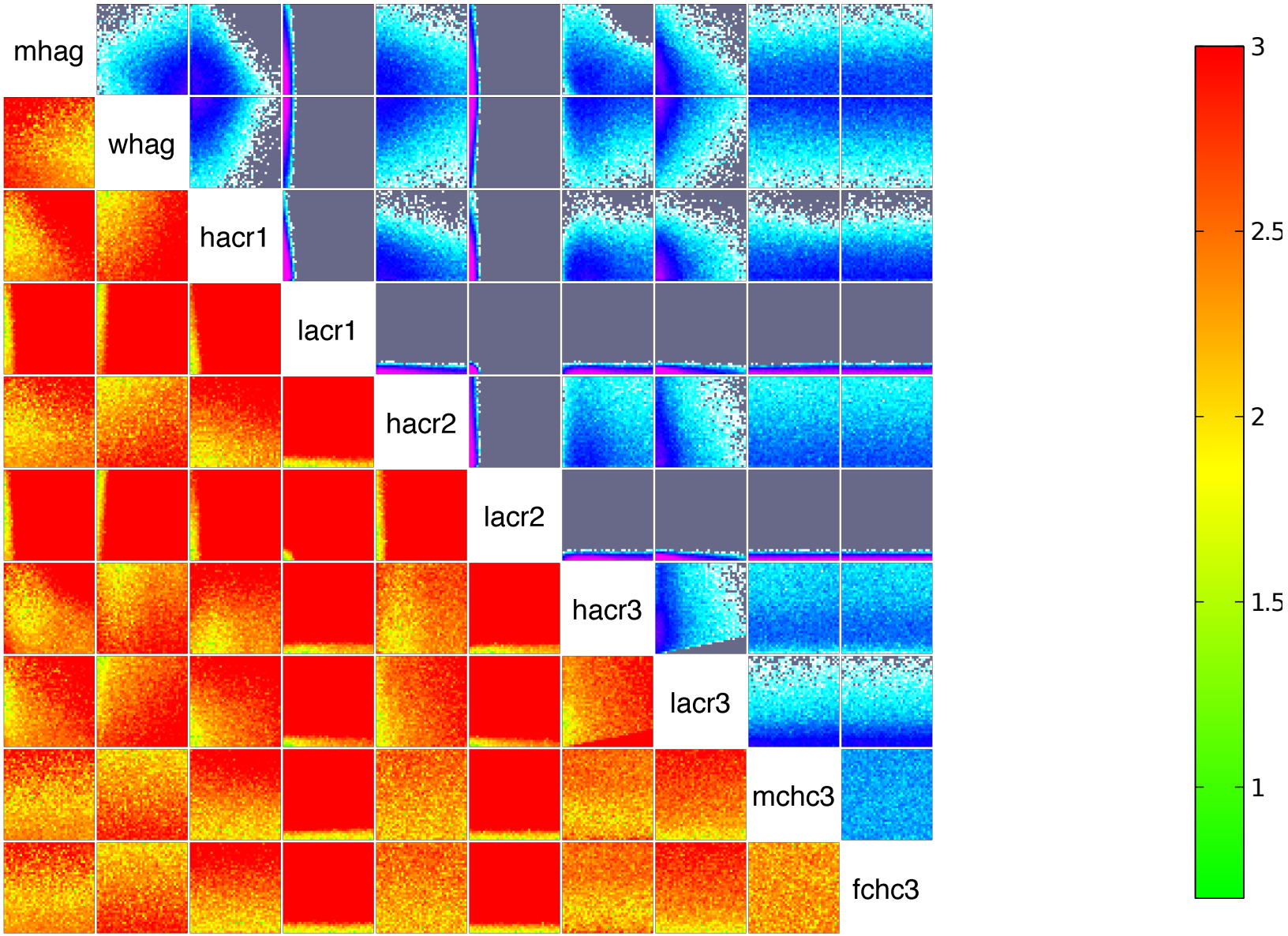
- History matching iterated through 10 waves, 200-500 model runs per wave

Wave 1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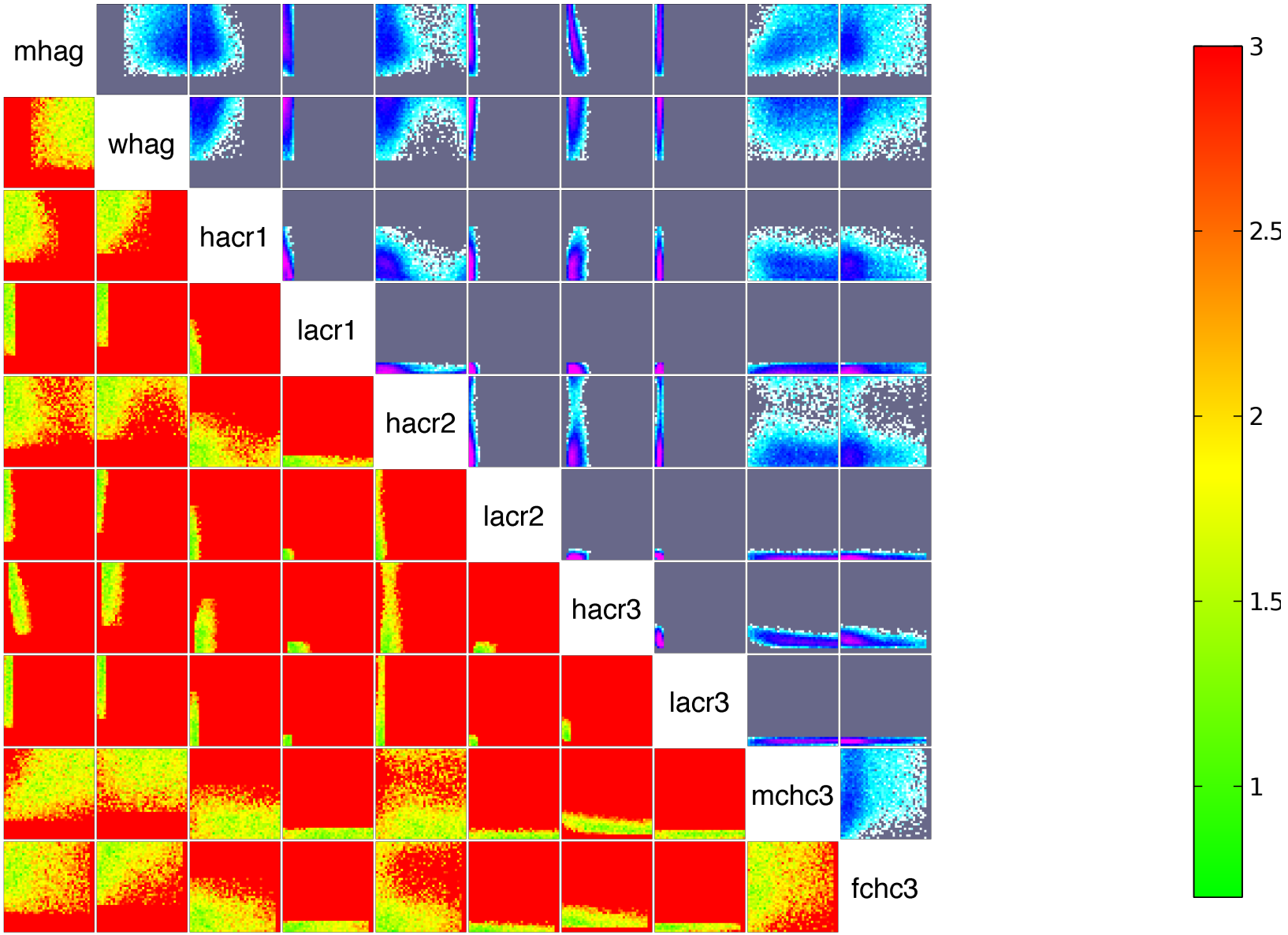
whag
hacr1
lacr1
hacr2
lacr2
hacr3
lacr3
mchc3
fchc3
3
2.5
2
1.5
1

Wave 4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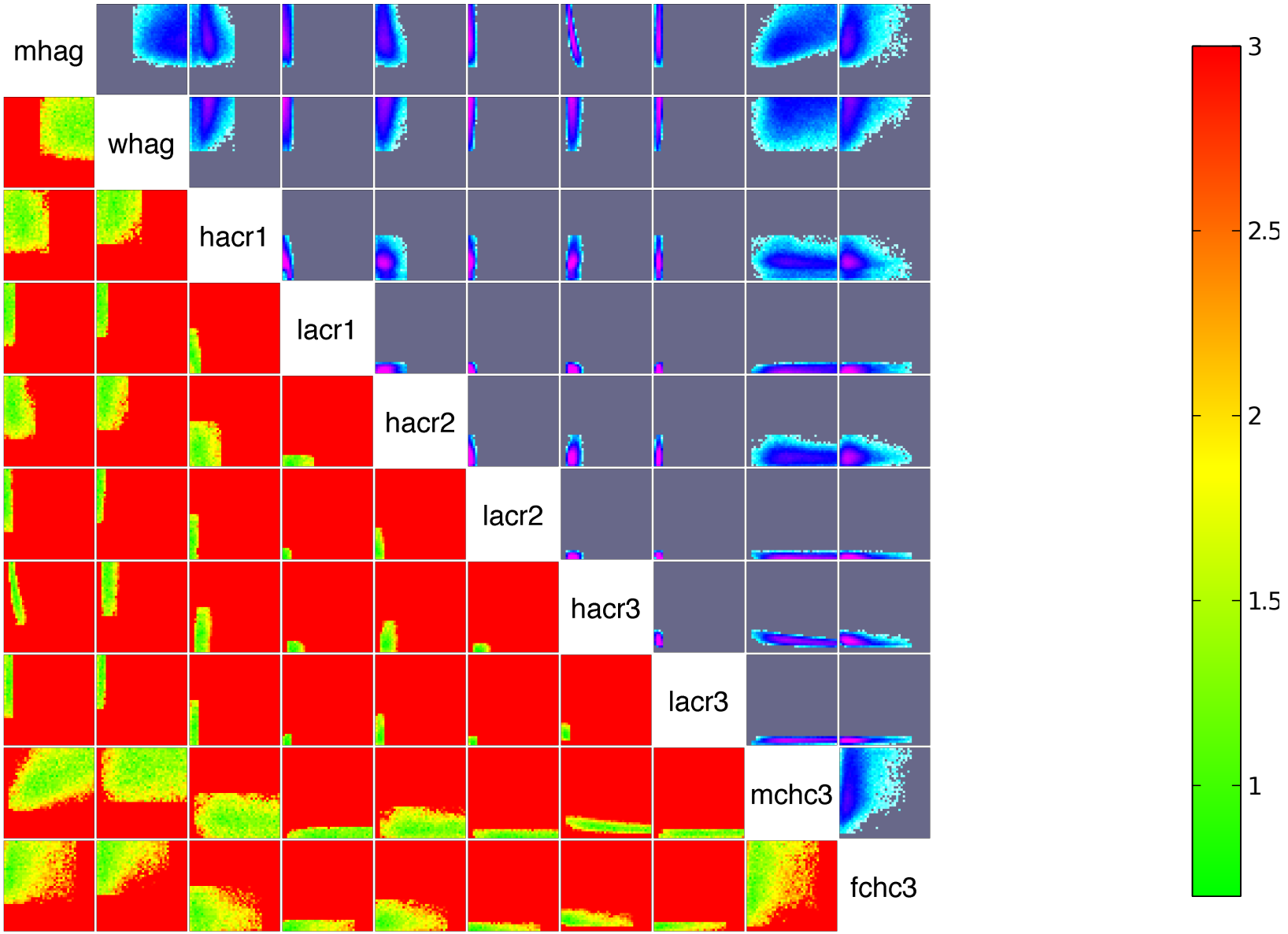
whag
hacr1
lacr1
hacr2
lacr2
hacr3
lacr3
mchc3
fchc3
3
2.5
2
1.5
1

Wave 7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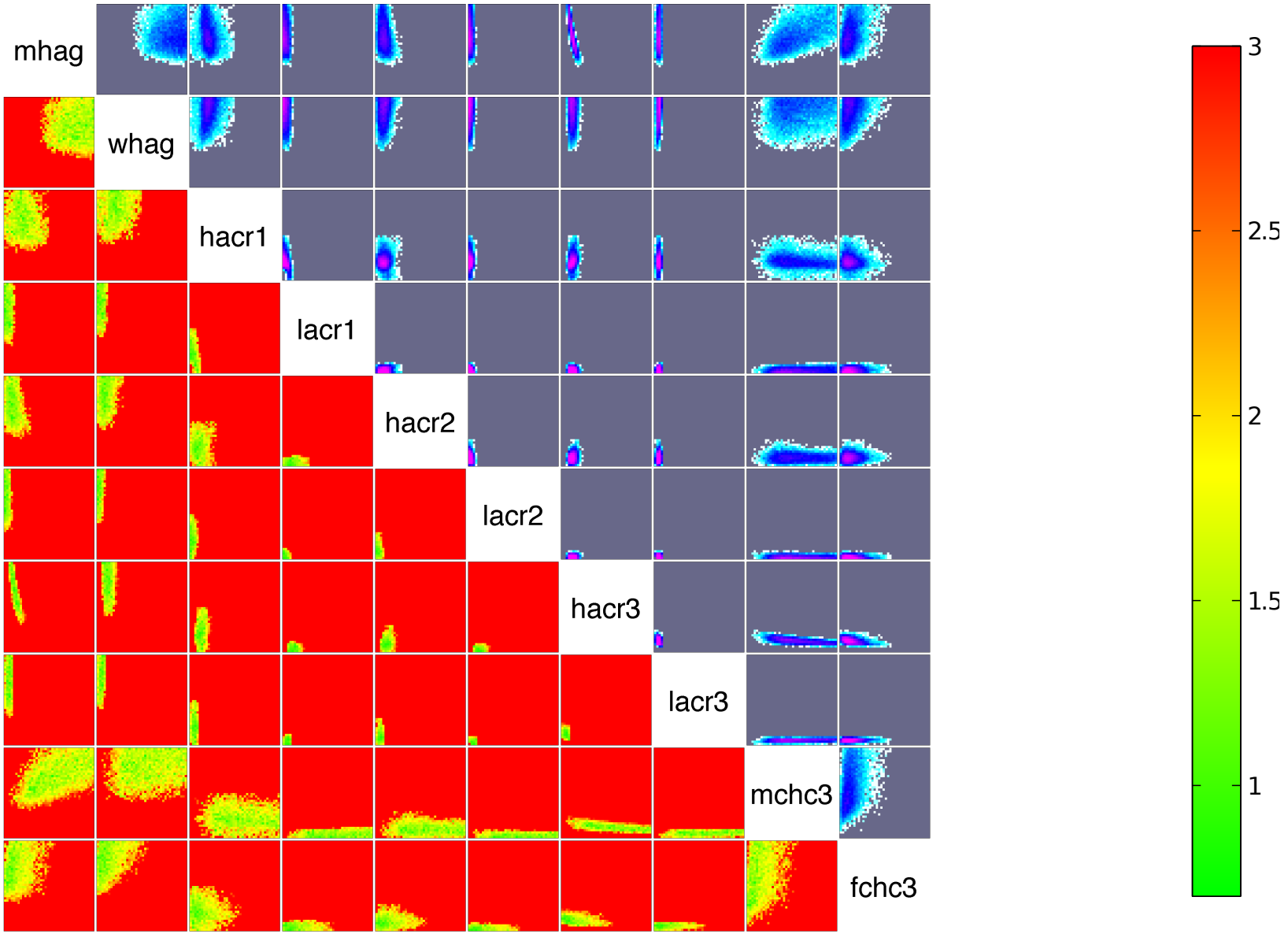
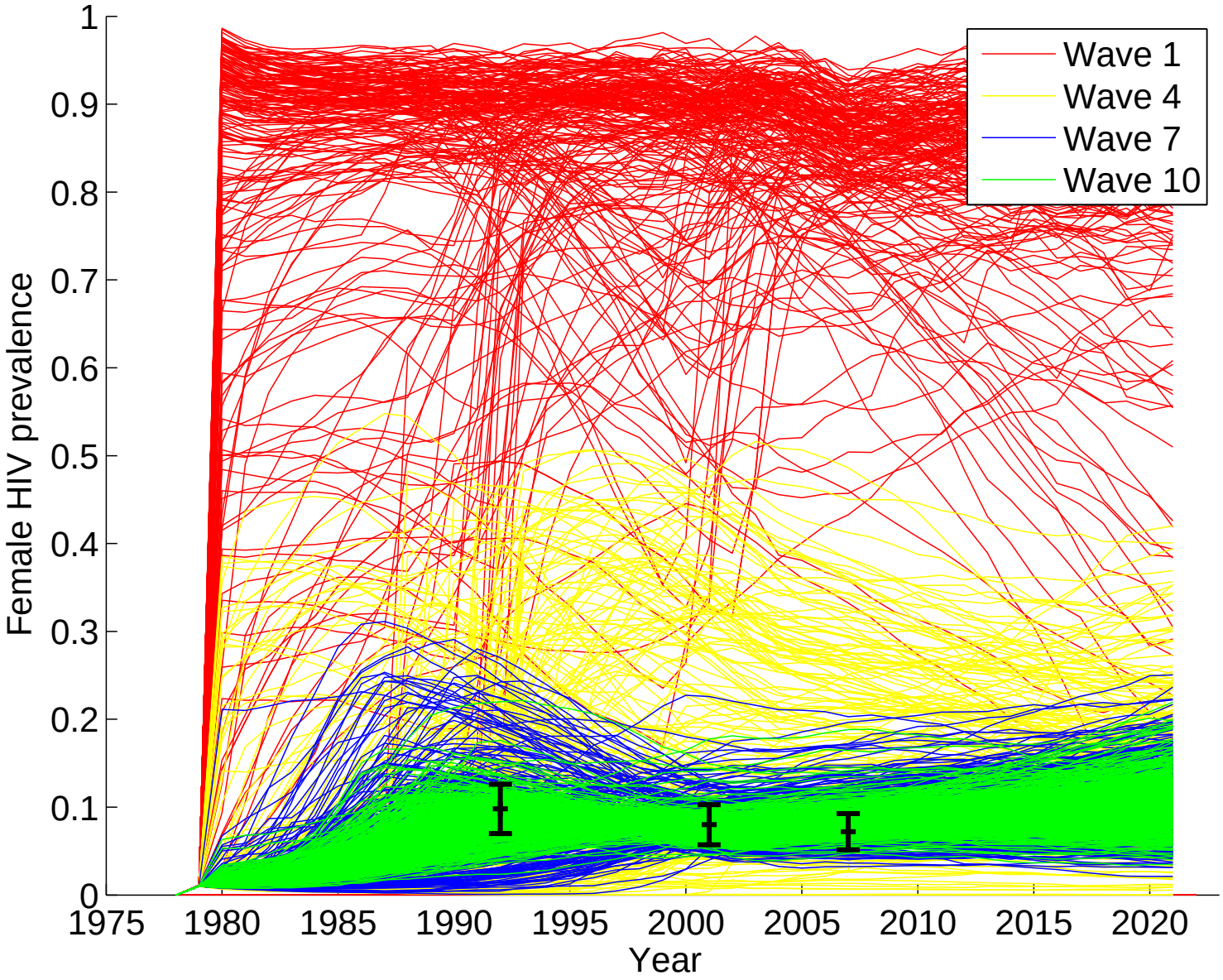
whag
hacr1
lacr1
hacr2
lacr2
hacr3
lacr3
mchc3
fchc3
3
2.5
2
1.5
1

Wave 9


mhag
whag
hacr1
lacr1
hacr2
lacr2
hacr3
lacr3
mchc3
fchc3
3
2.5
2
1.5
1

Model runs after history matching



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Further reading/papers at

`jeremy-oakley.staff.shef.ac.uk`